

# Vibration sensitivity optimization of a 30-cm-long high-finesse optical reference cavity by multi-body modelling

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**Abstract**—Ultra-stable laser has important applications in many scientific research fields. The limited frequency stability of the ultra-stable laser is restricted by the thermal noise of the optical reference cavity. Increasing the length of the optical reference cavity is one of the effective methods to reduce thermal noise, but at the cost of higher sensitivity to vibration. In this work, we focused on the vibration sensitivity optimization of our home-built 30-cm-long high-finesse optical reference cavity. By establishing a finite element multi-body model, including the vacuum chamber, three layers of the thermal shields, and U-shaped Zerodur mount. We calculated the vibration sensitivity of the 30-cm-long high-finesse optical reference cavity under various parameters. Finally, the optimal parameters of zero vibration sensitivity of the 30-cm-long high-finesse optical reference cavity are obtained.

**Keywords**—Ultra-stable laser, optical reference cavity, multi-body modelling, Vibration sensitivity

## I. INTRODUCTION

Ultra-stable laser is fundamental apparatus utilized in diverse applications ranging from fundamental scientific researches, such as precision spectroscopy [1], atomic optical clock [2], and gravitational wave detection [3], to industrial applications including laser gyroscopes [4], photonic microwave generation [5], coherent optical frequency dissemination [6] and so on. The architecture of ultra-stable laser typically involves precise laser frequency locking to a spectral resonance of a high-finesse optical reference cavity. The performance metrics of the stabilized laser, namely absolute linewidth and fractional frequency stability, is fundamentally restricted by its thermal noise limit of the cavity.

Reducing the thermal noise limit involves utilizing materials with low mechanical loss angle, minishing curvature of the cavity mirror, and increasing the length of the cavity spacer. Among these methods, the last is remarkably effective. However, the longer the length of an optical reference cavity is, the more difficult and tedious the optimization of vibration sensitivity becomes. Previous investigations by researchers, such as S. Häfner et al. [7] and Li Jin et al. [8], have yielded instructional results on the vibration sensitivity of the high-finesse optical reference cavity beyond 30 cm via single-body modelling. To date, multi-body modelling involving the whole cavity assembly is still unexplored. Theoretically, multi-body modelling provides more practical analysis of the assembly compared to single-body modelling. In the paper, we applied multi-body modelling to numerically analyse the

vibration sensitivity of a 30-cm-long high-finesse optical reference cavity.

## II. THE DESIGN OF THE HIGH-FINESSE OPTICAL REFERENCE CAVITY SYSTEM

The design of the high-finesse optical reference cavity is illustrated in Fig. 1, the 30-cm-long notched cylindrical spacer with a diameter of 150 mm is made from ultra-low expansion (ULE) glass. The mirror substrate with a diameter of 25.4 mm and a thickness of approximately 6.3 mm is fused silica. The curvature radius of one cavity mirror is set to 1 m, and the other one is plane. Two identical ULE-glass annuluses optically contacted to the rear side of the both mirror substrates are used to avoid the effect of the differential thermal expansion between the fused silica substrate and the ULE-glass spacer. The thickness is approximately 6.3 mm, and the diameter of the through hole is 9 mm. A 4 mm-diameter vent is drilled in the middle of the cavity spacer for vacuum exhaust. Four Viton hemispheres hold the high-finesse optical reference cavity.

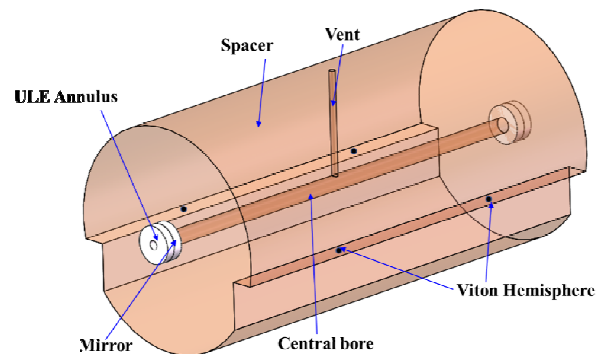


Fig. 1. Schematic diagram of the high-finesse optical reference cavity

As shown in Fig. 2, the 30-cm-long high-finesse optical reference cavity system is mainly assembled by a vacuum chamber, three layers of thermal shields, a U-shaped support and the optical reference cavity described above. From the inside out, the 30-cm-long high-finesse optical reference cavity is horizontally resting on four points of a U-shaped mount made by Zerodur. Between the Zerodur support and the bottom plate of the third layer of the thermal shield, three glass balls serve as thermal isolation and self-balancing mount to overcome different supporting forces from the above load caused by imperfect fabrication. To increase the time constant of the thermal isolation, three layers of the thermal shields made by

copper are applied. the surfaces of each thermal shield are gold-plated to reduce thermal emissivity. Two apertures on two side plates of every thermal shield is drilled for resonated laser beam. Between all the layers of the thermal shields and the vacuum chamber, this three-point mounting is also employed, but the spacers are changed to glass

columns. The aluminum-alloy vacuum chamber is sealed with a rubber ring. A ion pump at a rate of 75 L/s is connected onto the chamber. The whole high-finesse optical reference cavity system is placed on an active vibration-isolated platform.

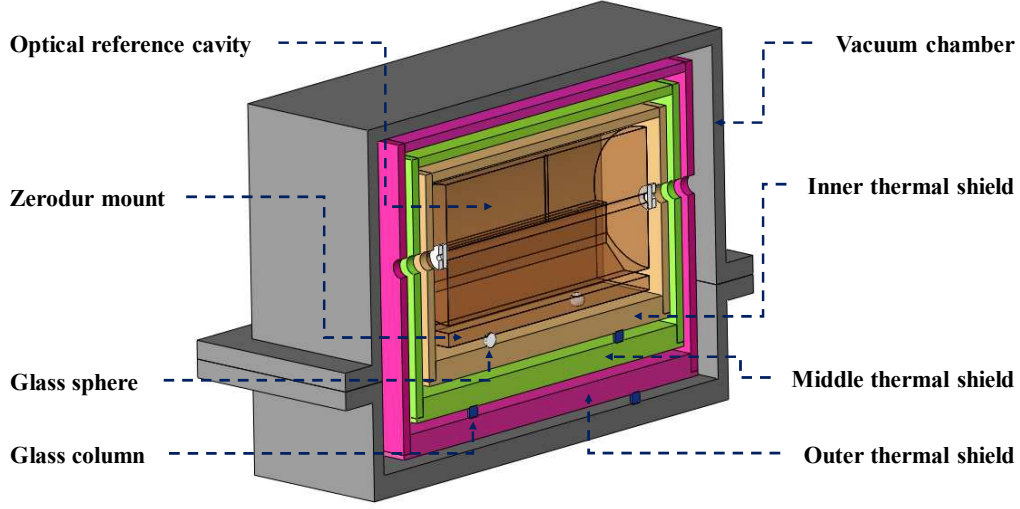


Fig. 2. Section view of the 30-cm-long high-finesse optical reference cavity system.

TABLE I. THE PHYSICAL PROPERTIES OF MATERIALS APPLIED IN MULTI-BODY MODEL OF THE HIGH-FINESSE OPTICAL REFERENCE CAVITY SYSTEM

Materials	ULE	Fused Silica	Zerodur	Viton	copper	Aluminium
Density (kg/m <sup>3</sup> )	2210	2203	2530	1900	8930	2700
Young's Modulus (GPa)	67.6	72.7	91	0.0008	110	69
Poisson Ratio	0.16	0.17	0.24	0.27	0.37	0.33

### III. NUMERICAL CALCULATION

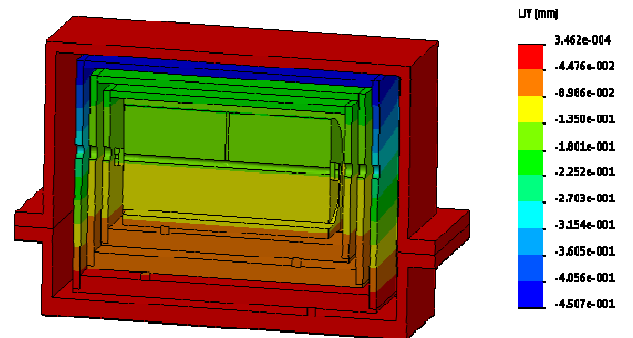
In general, the vibration sensitivities of the optical reference cavity along the three directions can be expressed as [9-12]

$$\begin{cases} S_x = \frac{\Delta L_x}{Lg} \\ S_y = \frac{\Delta L_y}{Lg} \\ S_z = \frac{\Delta L_z}{Lg} \end{cases} \quad (\text{Eq.1})$$

where  $S_x$ ,  $S_y$ , and  $S_z$  are the vibration sensitivities of the optical reference cavity along the x-axis, y-axis, and z-axis, respectively.  $\Delta L_x$ ,  $\Delta L_y$ , and  $\Delta L_z$  denote the variation of the optical reference cavity length in the optical axes caused by the acceleration along the x-axis, y-axis, and z-axis, respectively.  $L$  represents the length of the optical reference cavity.  $g$  denotes gravitational acceleration.

Table 1 showcases the properties of materials applied in the numerical simulation of the vibration sensitivity of the whole high-finesse optical reference cavity system. When the bottom part of the vacuum chamber is fixed, the displacement along the optical axis of the optical reference cavity can be observed in Fig. 3(a). Generally, the displacement along the optical axis is on the order of 0.1

mm. The vibration sensitivity of the optical reference cavity is plotted in Fig. 3(b). The calculations present that the vibration sensitivity reaches a null value when the distance between the hemisphere and the near end of the optical reference cavity is around 48.5 mm, although the radius of the Viton hemispheres ranges from 1.5 mm to 5 mm.



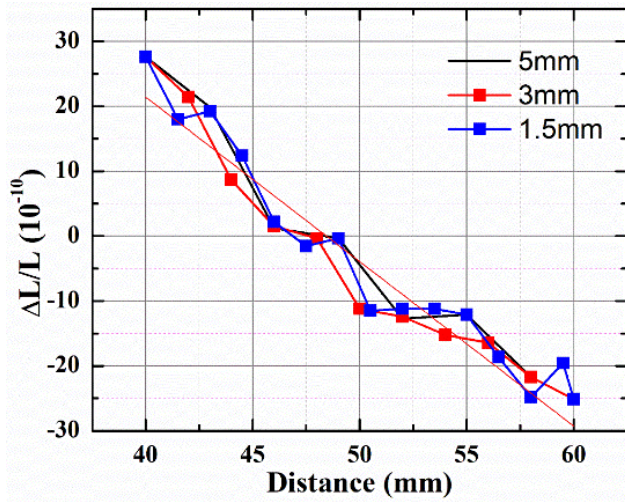


Fig. 3. Numerical calculation of the vibration sensitivity of the 30-cm-long high-finesse optical cavity system. (a) The displacement along the axis. (b) The vibration sensitivity of the cavity assembly. The black line denotes the radius of the Viton hemispheres is 10 mm, while the red line with squares is 6 mm, and the blue line with square is 3 mm. The solid red line is linear fit of the vibration sensitivity.

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